# Utilizing Fuzzy Optimization for Distributed Generation Allocation

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*Abstract*— Distributed generation (DG) allocation problem is addressed utilizing fuzzy multi-objective optimization in this paper. It is shown that the methodology provides needed consideration for DG allocation and accounts for uncertainty using fuzzy set theory. Voltage drop reduction, short circuit capacity (SCC) augmentation, decrease operation cost and system losses reduction were considered as objectives for formulating fuzzy optimization. The paper discusses in detail the approach adopted and several numerical examples are presented to test the developed methodology.

*Index Terms*—distributed generation, distribution systems, fuzzy set theory, optimization.

## I. INTRODUCTION

ISTRIBUTED energy resource (DG) interconnection in electric distribution systems brings new dimension in planning and operations of these systems. The nature of distribution systems before distributed generation interconnection are passive and will be much active in the presence if such energy resource. Regulatory code [1] for distribution systems specify data for studying DG integration into distribution systems are detailed for the anticipation of such interconnection. The interconnection of DG, sometimes called as embedded or dispersed generation, accounts for detailed planning studies [2]. DG interconnection can impact profile of voltage, equipment loading, system reliability, stability, fault currents [3] and power quality in distribution systems [2], [4]. But these effects can vary depending on the placement and sizing of DG. Allocation of distributed DG in distribution systems has been a subject of research given this assumption.

Using non-linear optimization [5], optimal placement of DG has been identified in an IEEE 30 bus system which is a transmission test system. In [6]-[7], DG investment planning was optimized using technical and economic methods and consideration of electricity market parameters. Analytical approaches minimizing line losses were also utilized for DG allocation as provided in [8]. In [9], the authors have integrated DG in distribution systems using power systems studies coupled with linear programming method. Analyzing these studies, the consideration of uncertainty in the DG allocation in distribution systems is neglected. Papers [10]-[11] utilized evolutionary programming for identifying placement of DG in distribution systems. In [10], a multi-

E. B. Cano is with the System Operations Group, National Transmission Corporation, Philippines and Department of Electrical Engineering, Holy Angel University, Angeles City, Philippines (e-mail: ebcano@gmail.com). objective index was formulated to quantify the impact of DG integration. Authors in [11] introduced a cost based evolutionary programming method for placement of DG. In this reports, rigorous mathematical computational effort is required to arrive to the solution. A more simple and straightforward solution can be formulated.

In this paper, fuzzy multi-objective optimization as applied to DG allocation in distribution systems is discussed. Fuzzy sets theory can handle uncertainties and imprecision [12] and are much less in computational exertion.

This paper is organized as follows: Section I introduces the research subject area; Section II discusses the fuzzy optimization methodology applied to DG allocation; Section III presents several numerical examples; lastly, Section IV presents the conclusions and further recommendations of the paper.

## II. FUZZY OPTIMIZATION METHODOLOGY

### A. DG Allocation Considerations

1.

DG integration in distribution systems are expected to impact voltage drop, short circuit capacity (SCC), system losses and operational cost.

The DG will provide voltage support upon interconnection that the percent voltage drop is likely to decrease depending on the placement. Voltage drop analysis from load flow computations was employed here.

Voltage sags and fluctuations are mitigated by higher SCC within the distribution system which will be augmented by the DG integration. In this paper, we utilized grid positive sequence impedance as 0.144 + j1.4022 ohms and DG positive sequence impedance as 6.2972 ohms [10]. Short circuit analysis was utilized for SCC computation.

In the case of losses, the application of DG within load centers will decrease line losses for power delivery. Upon the operation of DG, operational cost for distribution system will be impacted for DG cost is expected to be lower than the grid price [6].

So, in this study the following multi-objective functions are considered:

Minimization of *PVD*  

$$PVD = \frac{V_{Si} - V_{Ri}}{V_{Si}} \ge 100$$
(1)

Where, *PVD* is percent voltage drop,  $V_{Si}$  is sending end line voltage and  $V_{Ri}$  is receiving end line voltage.



Figure 1. Membership function for percent voltage drop metric.

2. Maximization of SCC  

$$SCC = \sqrt{3} \square_{3F} \square k V_{base}$$
(2)

Where, SCC is short circuit capacity (MVA),  $I_{3F}$  is threephase fault at the nth bus and  $kV_{base}$  is line voltage base.

3. Minimization of system losses  

$$P_l = \operatorname{Re}(Z_{ij}) \square I_{ij}^2 \qquad (3)$$

Where,  $P_i$  is the total system loss in kW,  $Re(Z_{ij})$  is the resistance of line from node *i* to node *j*, (Ohms) and  $I_{ij}$  is current flow from node *i* to node *j*, (Amps.).

4. Minimization of operation cost  

$$C(P_i) = P_g \Box \lambda_g + P_{dg} \Box \lambda_{dg}$$
(4)

Where,  $C(P_i)$  is the cost function of power generated in the distribution system (\$/MW),  $P_g$  is the power generated from the grid, (MW),  $P_{dg}$  is the power generated by the DG, (MW),  $\lambda_g$  is the nodal price of power generated by the grid, electricity market price, we used 70\$/MWh [6], and  $\lambda_{dg}$  is the marginal cost of power generated by the DG, we used 42 \$/MWh [6].

The investment cost of putting up DG within the franchise of the distribution system is assumed equal at all possible system connection points, thus it is neglected in this study.

## B. DG Metrics Membership Functions Development

## 1) PVD Metric Membership Function

For each case, we classify the PVD metric as:

$$x_i = \frac{PVD_i}{PVD_0}$$
 for  $i = 1, 2, ...n$  (5)

Where  $x_i$  is the normalized percent voltage drop from the ratio of  $PVD_i$ , which is the PVD for case *i*, and  $PVD_o$ , which is the PVD for base case before DG interconnection. From equation (5), when  $x_i$  is high, the reduction of PVD is low after DG interconnection, then it is assigned with lower membership value. On the other hand, when  $x_i$  is low, the reduction of PVD is high then it is assigned with higher membership value. Then the PVD membership function, given in figure 1, is defined:



Figure 2. Membership function for short circuit capacity metric.

$$\mu_{PVD} = \begin{cases} 1 & for & x_i \le x_{\min} \\ \frac{x_{\max} - x_i}{x_{\max} - x_{\min}} & for & x_{\min} < x_i < x_{\max} \\ 0 & for & x_i \ge x_{\max} \end{cases}$$
(6)

It is assigned that  $x_{min} = 0.25$  while  $x_{max} = 1.0$  in this paper. This indicates that if the  $PVD_i$  is 25% or less of  $PVD_o$ , the unity membership value is assigned and if the  $PVD_i$  is 100% or more of  $PVD_o$ , the zero membership value is assigned. In this paper, PVD is computed at the node with highest voltage drop with reference to the grid or substation node in the simulation of the base case.

### 2) SCC Metric Membership Function

For each case, we classify the SCC metric as:

$$y_i = \frac{SCC_i}{SCC_0} \text{ for } i = 1, 2, ...n$$
 (7)

Where  $y_i$  is the normalized short circuit capacity from the ratio of  $SCC_i$ , which is the SCC for case i, and  $SCC_o$ , which is the SCC for base case before DG interconnection. From equation (7), when  $y_i$  is high, the increase in SCC is high after DG interconnection, then it is assigned with higher membership value. On the other hand, when  $y_i$  is low, the increase in SCC is low then it is assigned with lower membership value. Then the SCC membership function, given in figure 2, is defined:

$$\mu_{SCC} = \begin{cases} 0 & \text{for } y_i \leq y_{\min} \\ \frac{y_{\max} - y_i}{y_{\max} - y_{\min}} & \text{for } y_{\min} < y_i < y_{\max} \\ 1 & \text{for } y_i \geq y_{\max} \end{cases}$$
(8)

It is assigned that  $y_{min} = 1.0$  while  $y_{max} = 1.80$  in this paper. This signifies that if the  $SCC_i$  is 100% or less of  $SCC_o$ , the zero unity membership value is assigned and if the  $SCC_i$  is 180% or more of  $SCC_o$ , the unity membership value is assigned. In this paper, SCC is computed at the node with highest voltage drop in the simulation of the base case.



Figure 3. Membership function for system loss metric.



Figure 4. Membership function for operation cost metric.

Where  $z_i$  is the normalized system loss index from the ratio of  $P_{li}$ , which is the  $P_l$  for case *i*, and  $P_{lo}$ , which is the  $P_l$  for base case before DG interconnection. From equation (9), when  $z_i$  is high, the reduction of  $P_l$  is low after DG interconnection, then it is assigned with lower membership value. On the other hand, when  $z_i$  is low, the reduction of  $P_l$  is high then it is assigned with higher membership value. Then the  $P_l$  membership function, given in figure 3, is defined:

$$\mu_{P_l} = \begin{cases} 1 & \text{for} & z_i \leq z_{\min} \\ \frac{z_{\max} - z_i}{z_{\max} - z_{\min}} & \text{for} & z_{\min} < z_i < z_{\max} \\ 0 & \text{for} & z_i \geq z_{\max} \end{cases}$$
(10)

It is assigned that  $z_{min} = 0.25$  while  $z_{max} = 1.0$  in this paper. This indicates that if the  $P_{li}$  is 25% or less of  $P_{lo}$ , the unity membership value is assigned and if the  $P_{li}$  is 100% or more of  $P_{lo}$ , the zero membership value is assigned.

#### 3) Operation Costs Metric Membership Function

For each case, we classify:

$$w_i = \frac{C(P_i)}{C(P_0)} \text{ for } i = 1, 2, \dots n$$
 (11)

Where  $w_i$  is the normalized operation cost from the ratio of  $C(P_i)$ , which is the C(P) for case *i*, and  $C(P_o)$ , which is the C(P) for base case before DG interconnection. From equation (11), when  $w_i$  is high, the reduction of C(P) is low after DG interconnection, then it is assigned with lower membership value. On the other hand, when  $w_i$  is low, the reduction of C(P) is high then it is assigned with higher membership value. Then the C(P) membership function, given in figure 4, is defined:

$$u_{C(P)} = \begin{cases} 1 & \text{for } w_{i} \leq w_{\min} \\ \frac{w_{\max} - w_{i}}{w_{\max} - w_{\min}} & \text{for } w_{\min} < w_{i} < w_{\max} \\ 0 & \text{for } w_{i} \geq w_{\max} \end{cases}$$
(12)

It is assigned that  $w_{min} = 0.50$  while  $w_{max} = 1.0$  in this paper. This indicates that if the  $C(P_i)$  is 50% or less of  $C(P_o)$ , the unity membership value is assigned and if the  $C(P_i)$  is 100% or more of  $C(P_o)$ , the zero membership value is assigned.

## C. Max-min Fuzzy Optimization

The fuzzy multi-objective optimization methodology is one algorithm that can be proposed.

The following are the specific steps for this solution methodology.

- Step 1 For each node of possible DG interconnection, all the membership functions discussed above are evaluated.
- Step 2 The scale of overall satisfaction for a specific node of DG possible interconnection is the minimum of all the above membership functions.

$$N_{i,n} = \min(\mu_{PVD}, \mu_{SCC}, \mu_{P_i}, \mu_{C(P)}) \text{ for } i = 1, 2, ...n$$
(13)

Step 3 - The optimal solution of the DG allocation is the maximum of the scales of overall satisfaction.

$$OPS_i = \max(N_{in}) \tag{14}$$

#### **III. NUMERICAL EXAMPLES**

We consider radial and meshed distribution systems from paper [8] to test the methodology presented herein and compare its results with [8]. Distribution data can be acquired from [8].

#### A. Radial Distribution System

For testing the method in radial distribution systems, we consider figure 5, with three cases of loading; uniformly distributed, centrally distributed and increasingly distributed. In this case, *PVD* and *SCC* are evaluated at node 11.

The methodology described above is followed and in Table 1, the results and comparison are presented.

The fuzzy optimization results shown in table 1 considers not only line losses as in the case of [8] but integrates fault currents, cost of operation and voltage drop reduction in the integration of DG in distribution systems. In this system, fuzzy optimization agrees only when the load is centrally distributed with the results in [8].



Figure 5. Radial distribution system for DG allocation [8].

 TABLE 1

 DG Allocation Results for Radial Distribution System.

Loading	Analytical Optimal DG Allocation [8]	Fuzzy Optimization DG Allocation
Uniform	6	7
Central	6	6
Increasing	8	9

## B. Meshed Distribution System

The figure below is a meshed distribution system from [8]. In this case, *PVD* and *SCC* are evaluated at node 3. We consider 5 MW DG allocation in this system and compare the results of [8] with the fuzzy optimization described herein. Table 2 presents the results for DG allocation.

It is shown in table 2 that the fuzzy optimization considering increased fault currents, reduced operation cost, reduction of voltage drop and system losses results in DG allocation in bus number 3 for the system given in figure below. The results of [8] agree with the results given here though line losses are only considered in [8].



Figure 6. Meshed distribution system for DG allocation [8].

 TABLE 2
 DG Allocation Results for Meshed Distribution System.

Meshed System	Analytical Optimal DG Allocation [8]	Fuzzy Optimization DG Allocation
Bus Number	3	3

## IV. CONCLUSIONS

In this paper, a fuzzy multi-objective optimization is developed for DG allocation for distribution systems. Detailed discussion of the methodology is provided and numerical examples are given where the said methodology is tested and compared with analytical algorithm. It was shown that the fuzzy optimization has technical and economic considerations neglected in other studies. For further refinement of the study, DG dispatch and demand variations should be integrated in the formulation. The sizing of DG can be included in the formulation for further research. Nevertheless, the formulation presented in this work is a useful tool for distribution system planning and operations engineers studying DG allocation in distribution systems.

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